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DD/S&T# 561-734

STAT

Dr. Dimitri S. Bugnolo

Dear Dr. Bugnolo:

Mr. Helms has passed on to this office your paper. Thank you very much for sending it. I will see that the technical people concerned in this matter get the benefit of its contents.

Sincerely,

Donald H. Steininger
Assistant Deputy Director
for
Science and Technology

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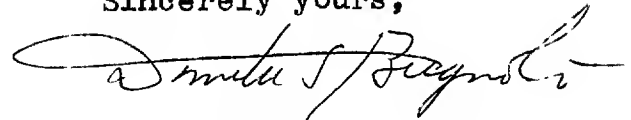72-1460

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March 8, 1972

DD/S&T# 560-72

Mr. Richard Helmes
Director
Central Intelligence Agency
Washington, D.C. 20505

Dear Mr. Helmes:

Enclosed is a copy of a paper of mine which would be of
considerable interest to the scientific people concerned with
missile versus decoy discrimination at low altitudes, if most
of these people were of sufficient competance.

By the way, have you ever discussed my case with Dr. Albert
Wheelon who is presently with the Hughes Aircraft Company, at
Culver City, California?

Sincerely yours,

Dr. Dimitri S. Bugnolo

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Anomalously Large Scattering by a Turbulent, Weakly Ionized Plasma

Dimitri S. Bugnolo
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(Received 10 May 1971)

An anomalously large result is predicted in the first Born approximation for the scattering cross section of a turbulent, weakly ionized plasma for the case when $\omega > \omega_p$, $\nu_{en} > \omega_p$, and $\omega \sim \nu_{en}$.

We have studied the effects of large-scale electron-neutral and neutral-neutral correlations on the electromagnetic cross section $\sigma(\vec{K})$ of a turbulent, weakly ionized plasma. A simplified case was reported previously and the theoretical methods have recently been reviewed.[1] It is convenient to compare this new result with the well-understood case, i.e., $\omega \gg \omega_p$, $\omega \gg \nu_{en}$, for which the scattering cross section is given by

$$\sigma_0(\vec{K}) = r_e^2 \langle n_e^2 \rangle E_{ee}(\vec{K}),$$

(1)

where ω_p is the electron plasma frequency; ν_{en}, the electron-neutral elastic collision frequency; r_e, the classical electron radius; $\langle n_e^2 \rangle$, the mean squared value of the electron density fluctuation due to turbulence; and $E_{ee}(\vec{K})$ is the spacewise spectrum of the electron fluctuations, defined by

$$E_{ee}(\vec{K}) = \int_{-\infty}^{\infty} C_{ee}(\vec{R}) e^{i\vec{K}\cdot\vec{R}} d^3R$$

(2)

with

$$C_{ee}(\vec{R}) = \langle n_e(\vec{r}) n_e(\vec{r}+\vec{R}) \rangle / \langle n_e^2 \rangle .$$

(3)

This was first studied theoretically by Villars and Weisskopf.[2] Experimental confirmation has been reported by Granatstein,[3,4] with some comments by Bugnolo.[5]

The effects of electron-neutral and neutral-neutral correlations on the cross section may be considered as follows. When $\omega > \omega_p$ with $\nu_{en} \sim \omega$, the effective dielectric constant of the plasma may be written as (mks)

$$\epsilon = \epsilon_0 \left[1 - \left(\frac{\omega_p}{\omega}\right)^2 \frac{1 + i\nu_{en}/\omega}{1 + (\nu_{en}/\omega)^2} \right],$$

(4)

with

$$\omega_p^2 = N_e e^2 / m_e \epsilon_0$$

and[6]

$$\nu_{en} = 2.82 \times 10^{-21} N_n P_c \langle u \rangle,$$

(5)

where N_n is the number density of the neutrals, P_c the probability of collision, and $\langle u \rangle$ the average *kinetic* velocity of the electrons. Taking

$$\omega_p^2 = \langle \omega_p^2 \rangle + \Delta\omega_p^2, \quad N_e = \langle N_e \rangle + n_e, \quad \nu_{en} = \langle \nu_{en} \rangle + \Delta\nu_{en}, \quad N_n = \langle N_n \rangle + n_n$$

(6)

in Eq. (4) yields the following result for the mean and fluctuating components of ϵ:

$$\langle \epsilon \rangle \cong \epsilon_0 \left\{ 1 - \frac{\langle \omega_p^2 \rangle}{\omega^2} \frac{1 + i\langle \nu_{en} \rangle/\omega}{1 + \langle \nu_{en} \rangle^2/\omega^2} \right\};$$

(7)

$$\Delta\epsilon \cong -\frac{4\pi r_e}{\mu_0 \omega^2} \left\{ n_e(\vec{r}) \left[1 + i\frac{\langle \nu_{en} \rangle}{\omega} \right] + i \frac{\langle N_e \rangle}{\langle N_n \rangle} \frac{\langle \nu_{en} \rangle}{\omega} n_n(\vec{r}) \right\} \left\{ 1 + \frac{\langle \nu_{en} \rangle^2}{\omega^2} \right\}^{-1}.$$

(8)

In the absence of polarization effects, the wave equation for the electric field intensity is

$$\nabla^2 \vec{E} + \omega^2 \mu_0 [\langle \epsilon \rangle + \Delta\epsilon(\vec{r})] \vec{E} = 0.$$

(9)

Substituting Eqs. (7) and (8) into (9) produces the following result for the first Born approximation for

the scattering cross section per unit volume per unit solid angle:

$$\sigma(\vec{K}) = \left[1 + \frac{\langle \nu_{en}\rangle^2}{\omega^2}\right]^{-1} \left\{ r_e^2 \langle n_e^2 \rangle E_{ee}(\vec{K}) + 2 r_e^2 \langle n_e n_n \rangle \frac{\langle N_e \rangle}{\langle N_n \rangle} \frac{\langle \nu_{en}\rangle^2}{\omega} \frac{E_{en}(\vec{K})}{[1 + (\langle \nu_{en}\rangle^2/\omega^2)]} \right.$$

$$\left. + r_e^2 \langle n_n^2 \rangle \frac{\langle N_e \rangle^2}{\langle N_n \rangle^2} \frac{\langle \nu_{en}\rangle^2}{\omega^2} \frac{E_{nn}(\vec{K})}{[1 + (\langle \nu_{en}\rangle^2/\omega^2)]} \right\}, \quad \omega > \omega_p. \tag{10}$$

We note that the first term within the brackets is just $\sigma_0(\vec{K})$ of Eq. (1), and is due to the electron-electron correlation. The second term results from electron-neutral correlations while the third term arises from neutral-neutral correlations. The spacewise spectra for $E_{en}(\vec{K})$ and $E_{nn}(\vec{K})$ are defined in a manner similar to Eqs. (2) and (3). In obtaining the above result we have assumed that the Mach number of the turbulence is small and that the kinetic temperature of the electrons is locally constant independent of the turbulent motion (a gradient in the mean kinetic temperature is permitted). It should be noted that neutral gas density fluctuations have been observed in the argon experiments.[7]

The anomalous result, an increase in the scattering cross section as $\omega \rightarrow \nu_{en}$ from above, can best be illustrated by an example. We have applied the theory to the particular case of a turbulent, weakly ionized, argon plasma of the type previously reported in the literature.[3-5,7,8] For a Reynolds number of 6000 or more, both the electron and neutral-gas density fluctuations exhibit an inertial region.[7,8] For the pressure of 45 Torr used in the first of these experiments, the inertial region will exist for wave numbers $K \lesssim 10$ cm^{-1}. This value of K corresponds to a back-scattering frequency range of

$$K = 2k_0 = 4\pi f_0/c, \quad f_0 \lesssim 24 \text{ GHz}. \tag{11}$$

At this pressure, the elastic collision frequency is approximately 3.4 GHz.[3] If, in an experiment of this kind, the current in the discharge is adjusted so that the plasma frequency is small as compared to the collision frequency, then a scattering experiment could, in principle, be performed over a frequency range of about 1 to 20 GHz. Within this range, E_{ee}, E_{en}, and E_{nn} would be inertial. While the scattering cross section of this experiment would be somewhat less then that observed previously, it is within the limits of present techniques. It is also of importance to note that such an experiment would be the first to observe an effect of neutral fluctuations on the electromagnetic scattering at microwave frequencies. To date, such observations have been limited to lasers whose frequency of operation is such that the scattering of the light is completely dominated by the neutrals.[9]

In order to illustrate better what might be expected from such an experiment we apply Eq. (10) directly. We note, however, that any particular conclusion is in part philosophical since the correlation of the electron and neutral-gas density fluctuations has never been measured. While this correlation must be zero in a collisionless plasma, it seems evident that some correlation must exist in a plasma dominated by collisions. This correlation might well be restricted to certain regions of the wave-number space that tend to move in unison.

For the sake of illustration, we have taken

$$\frac{\langle n_e^2 \rangle^{1/2}}{\langle n_n^2 \rangle^{1/2}} \cong \frac{\langle N_e \rangle}{\langle N_n \rangle}, \quad \langle n_e n_n \rangle \cong \langle n_e^2 \rangle^{1/2} \langle n_n^2 \rangle^{1/2}, \tag{12}$$

which would be representative of a form of correlated mixing. In this case Eq. (10) for $\sigma(\vec{K})$ reduces to

$$\sigma(\vec{K}) \sim \frac{r_e^2 \langle n_e^2 \rangle E(\vec{K})}{[1 + \langle \nu_{en}\rangle^2/\omega^2]} \left\{ 1 + 3 \frac{\langle \nu_{en}\rangle^2}{\omega^2} \frac{1}{[1 + \langle \nu_{en}\rangle^2/\omega^2]} \right\}. \tag{13}$$

We have plotted this result as Fig. 1, together with σ_0, as given by Eq. (1).

From inspection of this result, we note that the cross section $\sigma(\vec{K})$ *increases* rather then decreases as $\omega \rightarrow \nu_{en}$ from above. This leads to an anomalously large cross section of 1.3 at $\omega = 1.4\nu_{en}$. The dashed curve in Fig. 1 is the result that would have been obtained had the effects of electron-neutral and neutral-neutral correlation been neglected, namely

$$\sigma^1(\vec{K}) = \sigma_0 [1 + \langle \nu_{en}\rangle^2/\omega^1]^{-1}. \tag{14}$$

The theoretical results may also be applicable to the calculation of the radar cross section of the

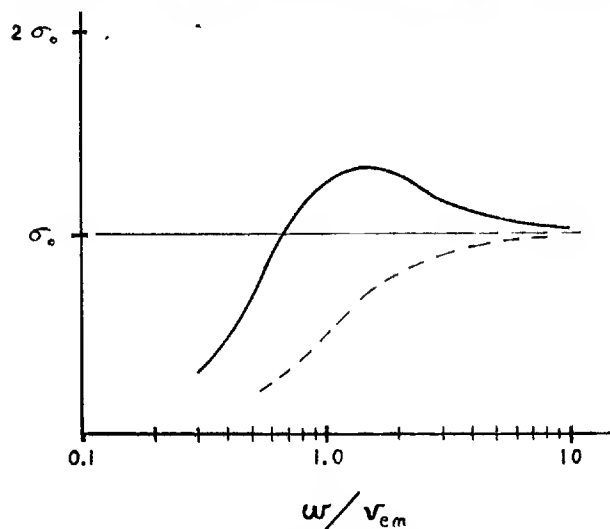wakes of hypersonic vehicles upon re-entry into Earth's atmosphere.

FIG. 1. The frequency dependence of the scattering cross section. Solid line: complete theory, Eq. (13). Dashed line: electron-electron correlations only, Eq. (14).

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